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Stability of the Most Hazardous Mars-Crossers

by

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ABSTRACT

The equations of motion of 4190 Mars Crossers (MCs) were numerically integrated to analyze their all possible close approaches to planets in the next 10^4 years. A sample of asteroids potentially hazardous to Mars was selected and properties of their chaotic motion on larger time scales were determined.

For samples of MCs closely approaching Mars, their mean frequency of close encounters was computed. We also analyzed the presence of mean motion and secular resonances.

The population of asteroids hazardous to Mars was found and the influence of frequent close approaches and resonances on the stability of their trajectories was estimated.

We also estimated the correlation between the frequency of close approaches to Mars and the Lyapunov Time (LT) of these asteroids. Some results concerning the correlation between mean motion/secular resonances and LT were also presented as well as three selected examples of dynamically interesting MCs.

Key words: *Minor planets, asteroids*

1. Introduction

In the last decade, the number of asteroids known as Mars Crossers (MCs) has significantly increased. Some of these small bodies, especially from the Amor group can also approach the Earth. Dynamical properties of MCs are different due to their large variety of distribution of their orbital elements. So far, the most comprehensive study of the MCs dynamics and evolution was presented by Michel *et al.* (2000).

In this paper we are mainly interested in effective MCs, *i.e.*, those that will closely approach Mars in the next 10^4 years. Initially, we regarded MC as a small body satisfying the following condition: 1.3 a.u. $< q < 1.67$ a.u.. Next we focused on searching for effective MC which in the near future will approach Mars at a distance smaller than the Martian Hill Sphere radius.

It is important to mention that there are other criteria of the classification of small bodies hazardous to planets such as the estimation of the minimum orbital intersection distance (MOID). This parameter is useful in classification of asteroids potentially hazardous to the Earth (PHA). However, small MOID value is not always related to the number and frequency of possible close approaches to a planet.

We decided to analyze all close approaches to planets of the available MCs orbital elements. Next, we chose specific candidates for effective MCs.

After initial selection of the sub-population of MCs, we numerically estimated their LTs. We also calculated the mean frequency of their close approaches to Mars. Frequent and regular close approaches to the planet influences the chaotic properties of minor body orbits. The quantitative estimation of this effect for our group of MCs was performed.

Furthermore, we checked the presence of mean motion and secular resonances in the MCs motion (the abbreviations MMR – mean motion resonance and SR – secular resonance will be used, respectively).

Some of MCs closely approaching Mars exhibit weak mean motion resonances in relation to Mars and inner planets. The presence of weak MMR plays an important role in the chaotic dynamics of MCs long-timescale migrations (Morbidelli and Froeschle 1999). For the previously chosen MCs group, we selected the "resonant" cases and estimated the influence of resonances on their stability.

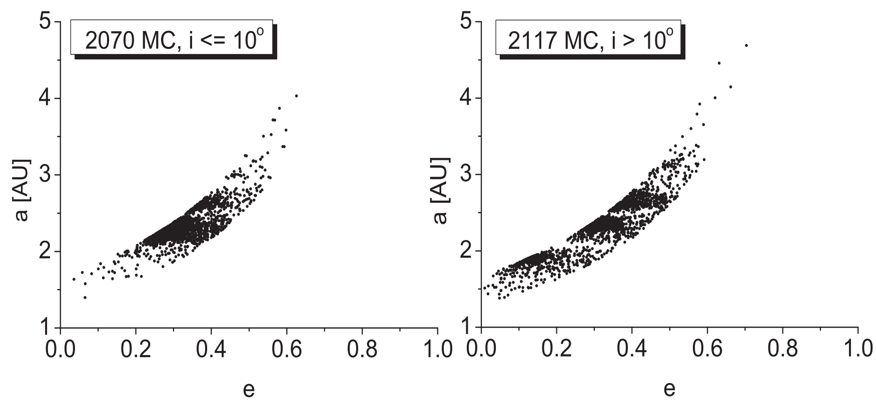

Fig. 1. The distribution of MCs with low ($i \leq 10^\circ$) and high ($i > 10^\circ$) inclinations. The number of MCs with small i values is relatively low in the vicinity of the Mars orbit.

2. Initial Data Selection

First, we selected initial orbits of planets and asteroids for numerical integration. Lowell Observatory database was a source of initial 4190 orbits (1.3 a.u. $<$ $q < 1.67$ a.u.). JPL DE405/406 (Standish 1998) and DE405/WAW Ephemeris (Sitarski 2002) were the sources of initial orbits of planets and the Moon.

To find all possible close approaches of MCs to planets in the near future we used the time-span of the next 10 000 years to analyze close approaches and resonances. In some cases the computations were extended to 20 000 years.

For additional analysis, especially of the LT, 1 Myr of numerical integration was required for the selected MCs.

3. Methods of Calculation

Methods based on packages SWIFT (Duncan, Levison and Lee 1998) and MERCURY (Chambers 1999) were used in the numerical integration of equations of the motion. Additional programs, useful for the estimation of the LT, determination of the time of stability, and the detection of resonances were written by the authors.

The Lyapunov Characteristic Exponents (LCEs) are a commonly used tool for quantitative estimation of chaos presence in dynamical systems. In most cases LCE values are connected with the stability and dynamical lifetimes. Due to the complexity and variety of dynamic behaviors of small bodies in the Solar System, this relation depends on a particular dynamical regime (Morbidelli and Froeschle 1996, Tancredi, Motta and Froeschle 2000).

The LTs of selected MCs were estimated numerically – as an inverse of LCEs at finite interval of time. LCE is defined as $\gamma = \lim_{t\to\infty}\chi(t)$, where $\chi(t) = \frac{1}{t}\ln\frac{d(t)}{d_0}$ is called Lyapunov Characteristic Indicator (LCI), d_0 is initial separation of two trajectories in six-dimensional space of orbital elements, $d(t)$ is final separation of trajectories after t time. Due to problems with practical application of the condition $t \to \infty$, all calculated and presented results are finite estimates of LCEs, based on partial results of numerical LCI calculations at finite interval of $t = 10^6$ yr.

In the two-particle method, the vector of separation is renormalized regularly after constant interval of time τ. In effect, this method has some limitations concerning the proper selection of parameters τ and d_0 (Tancredi, Sanchez and Roig 2001). These parameters must have optimal values, which allow to detect and determine the diveregence of nearby trajectories in the particular dynamic problem. In the motion of asteroids approaching inner planets, typical values of τ are of the order of 10 yr, and d_0 of the order of 10^{-8}. These limitations were taken into account during the calculations.

The presence of mean motion and secular resonances were analyzed during the numerical integration by tracking the values of mean motion, nodal and apsidal precession rate of MCs and planets. For our small group of effective MCs, we paid particular attention to the presence of weak MMR and SRs with Mars.

4. Orbital Evolution – Examples

4.1. Short-Time Evolution of 1999 KN15; MMR and Close Approaches

A typical example of an asteroid closely approaching Mars is 1999 KN15 (Fig. 2). Its orbit was determined from the short observational time-span (12 observations).

Table 1

The orbit of 1999 KN15

M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
The orbit for the epoch 1999 May 22.0 ET = JD 2451320.5					
7.°45422	2.4166592	0.3644022	94.°49337	116.°99452	3.°66743

Mean residual = 0.58 (MPC 35122, from 24 residuals)

In the first 5 000 yr in the future, the motion is dominated by 1 : 2 MMR with Mars (Fig. 2a) and shows a regular, shallow encounters with this planet (Fig. 2b). In the next 5 000 yr the influence of 1 : 2 MMR is smaller. The asteroid is in the different dynamical regime, with smaller inclination (Fig. 2c) and higher eccentricity (Fig. 2d). Additionally, more frequent and deeper close approaches to Mars are present. The current orbit of 1999KN15 and its orbital elements are presented on web page: *http://neo.jpl.nasa.gov/orbits/*.

4.2. Close Approaches to Mars of 2003 SE41

The 2003 SE41 asteroid exhibits the closest approach to Mars in our MCs sample. However it is based on a small number of observations. The new orbit was determined by G. Sitarski (private communication) from 35 observations (2003 September 17 – 2003 November 2) provided kindly by B. Marsden (Minor Planet Center, private communication) on October 6, 2004 (Table 2).

By integrating 1000 random "clone orbits" located in the vicinity of the nominal orbit, the closest possible approach to Mars was found on 2401, April 12.87457 UT at 0.00030637 a.u. *i.e.,* about 46 000 km or 12.5 Mars radii. Using the same orbits but another method of computations, Sitarski has computed 0.000284 a.u. as a minimum distance between Mars and asteroid. Thus his results are almost identical.

Due to the small amount of available short-arc observations, we expect that the predictions of close approaches to Mars of this object may change significantly

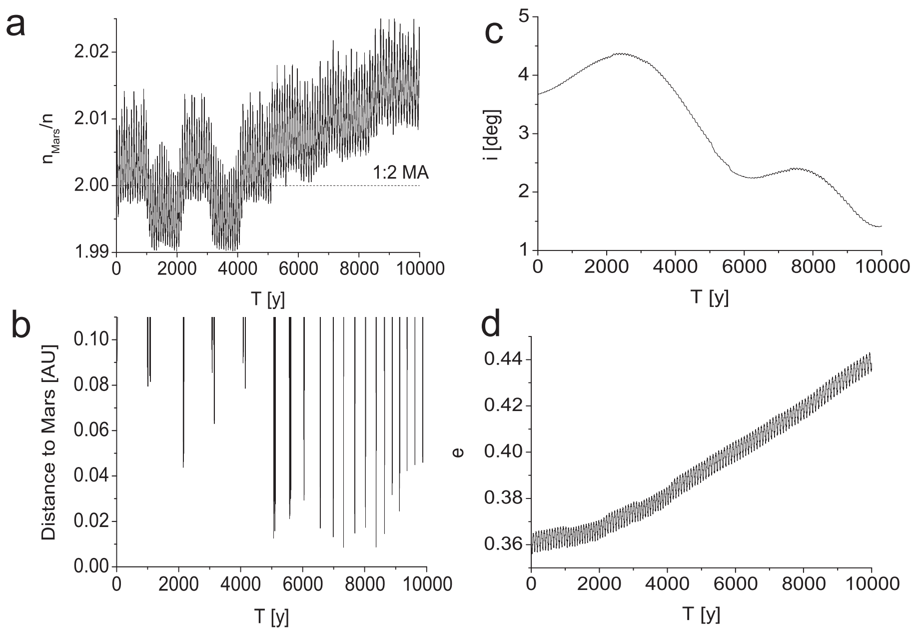

Fig. 2. The mean motion ratio (*a*), close approach range to Mars (*b*), inclination (*c*) and eccentricity (*d*) of 1999 KN15 in a short (10^4 yr) time-span.

Table 2

The orbit of 2003 SE41

M	*a* [a.u.]	*e*	ω_{2000}	Ω_{2000}	i_{2000}
The orbit for the epoch 2004 July 14.0 ET = JD 2453200.5					
46.°59628177	2.33842518643	0.35312620819	109.°18584070	329.°52457759	6.°60221096

Mean residual = 0.44 (from 70 residuals)

as a result of new observations and orbit determination/correction. Until the time of writing this paper (June 2006) there are no new observations of this asteroid. The presentation of orbit and orbital elements of 2003SE41 are available on NASA webpage (*http://neo.jpl.nasa.gov/orbits/*).

4.3. 2003 SC220; Coorbital Quasi-Satellite of Mars

The MC 2003 SC220 is temporary locked in MMR 1:1 with Mars. In Fig. 3a we can see the time evolution of its semi-major axis in the time-span of 10 000 years. It is clear that frequent, regular close approaches of the asteroid 2003 SC220 to Mars, as shown in Fig. 3b, are the cause of such behavior of the semi-major axis. The time of stability (TS) of this asteroid is about 9 000 years, as presented in Fig. 3c, so we can accurately predict the behavior of 2003 SC220 in this time-span (Włodarczyk 2001). Fig. 3d shows the time evolution of the orbits of the asteroid in a reference frame which co-rotates with Mars. Each orbit is separated in time by 5 000 days *i.e.*, about 137 years. From Fig. 3d it is clear that after about 700 years the asteroid 2003 SC220 will become a temporary moon of Mars. A similar situation will occur after 1420 years and will repeat regularly every 700 years.

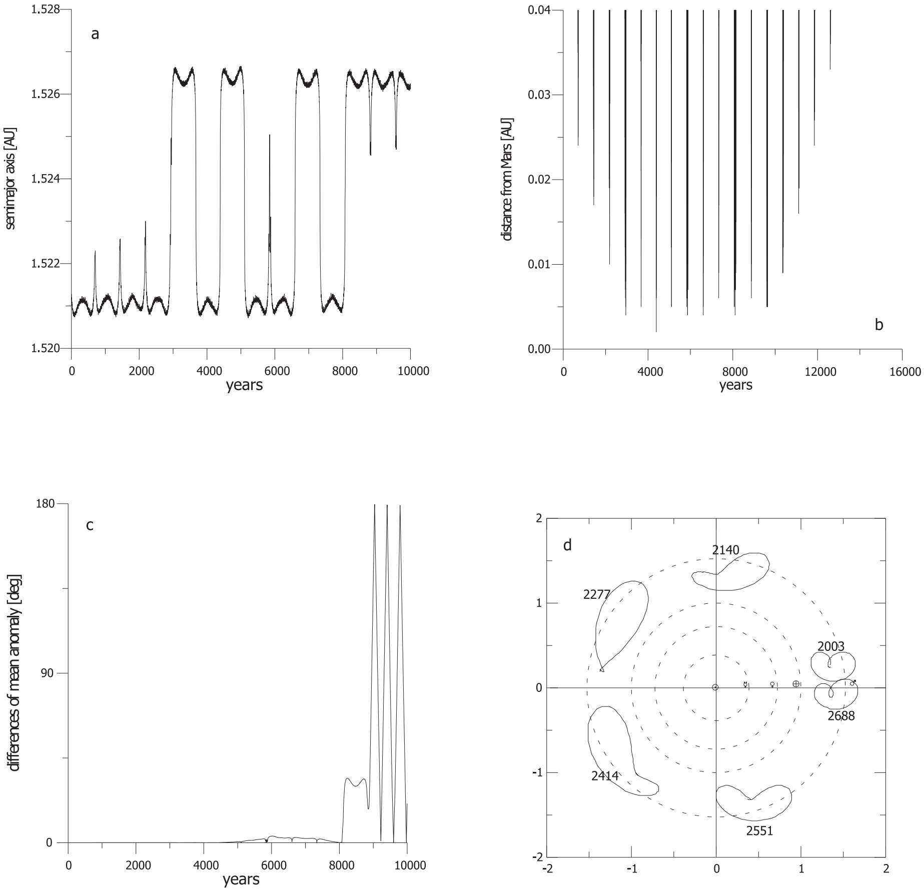

Fig. 3. Asteroid 2003 SC220 – the temporary trojan of Mars: *a* – time evolution of the semi-major axis, *b* – distance of the asteroid to Mars, *c* – differences of mean anomaly of the neighboring orbits, *d* – time evolution of the orbits in the reference frame rotating with Mars.

Table 3 lists the orbital elements of asteroid 2003 SC220. The orbital elements were taken from *astorb.dat* database of Lowell Observatory updated for JD 2453200.5 = 2004 July 14.0 TDT (*http://asteroid.lowell.edu/*).

The space orbit and orbital elements of 2003SE41 are also on the NASA page: *http://neo.jpl.nasa.gov/orbits/*.

Table 3

Starting orbital elements of the orbit 2003 SC220

M	a [a.u.]	e	ω_{2000}	Ω_{2000}	i_{2000}
The orbit for the epoch 2004 July 14.0 ET = JD 2453200.5					
176°372242	1.52133949	0.12436843	139°047894	199°285835	33°230367

5. Close Approaches of MCs to Mars and Other Planets

Table 4 shows all close approaches to planets found during initial selection of effective MCs. All close approaches closer than $3R_{\mathrm{Hill}}$ distance together with shallow encounters $d < 20R_{\mathrm{Hill}}$ to four inner planets and the Moon are also presented. We found 655 MCs approaching Mars at $d < 3R_{\mathrm{Hill}}$, but only 72 at $d < R_{\mathrm{Hill}}$ – selected for further analysis as effective MCs. The criterion of selection was at least one approach to Mars at $d < R_{\mathrm{Hill}}$ during next 10^4 yr. However, due to the limited number of $d < R_{\mathrm{Hill}}$ approaches, the ones with $d < 3R_{\mathrm{Hill}}$ were taken into account for the approach frequency analysis.

Table 4

The statistics of close approaches of 4190 MCs to planets in the next 10^4 yr.

	Mercury	Venus	Earth	Moon*	Mars	Jupiter	Saturn	Uranus
R_{Hill} [a.u.]	0.0015	0.0068	0.0100	0.0022	0.0072	0.3552	0.4356	0.4682
$3R_{\mathrm{Hill}}$ [10^6 km]	0.660	3.045	4.500	1.005	3.255	159.9	196.1	210.8
Approaches closer than $3R_{\mathrm{Hill}}$								
Number of approaches	1	4	65	3	4 967	8 267	139	10
Number of MCs	1	3	21	3	655	106	12	2
Approaches to inner planets closer than $20R_{\mathrm{Hill}}$								
$20R_{\mathrm{Hill}}$[a.u.]	0.0295	0.135	0.200	0.045	0.145			
Number of approaches	6	299	3 426	166	124 415			
Number of MCs	2	12	83	25	1 919			

* – R_{Hill} for small body-Moon-Sun system

The presence of regular close encounters to planets in the motion of a small body usually results in chaotic behavior. For effective MCs we expected to find the correspondence between the frequency of encounter to Mars and LT value. Similar studies for Jupiter Family Comets and selected NEA have already been presented by Tancredi (1998).

We estimated LT values for 72 MCs and presented them in Fig. 4 with mean approach frequency values.

Only 12 MCs have LTs smaller than 100 yr. For this, “most chaotic” group there are usually a few close approaches to Mars during 10^4 yr interval. Most LT values are of the order of 10^2 yr and for this group the highest LT values are connected with the low approach frequency.

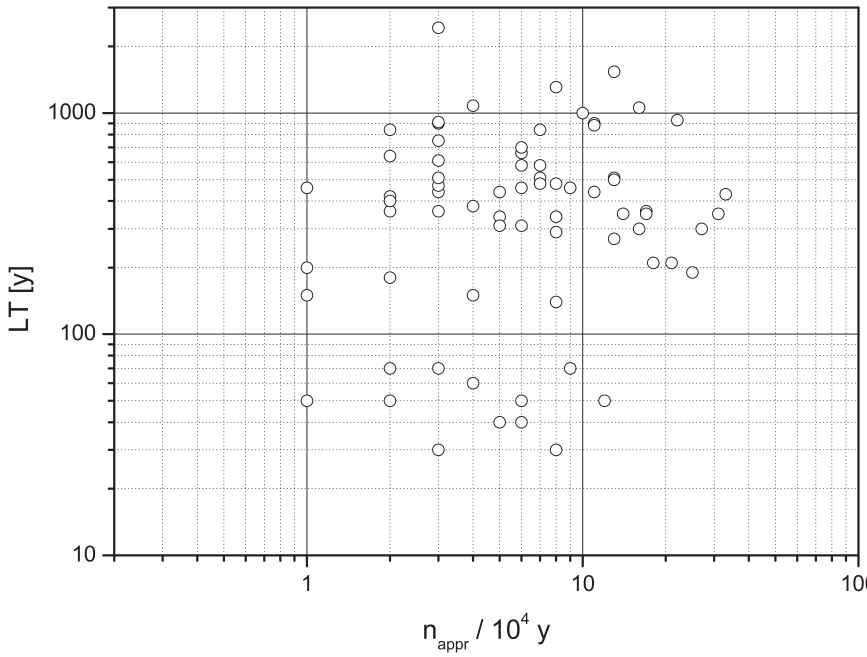

Fig. 4. LT values *vs.* mean approach frequency (the number of close approaches at $d < 3R_{\rm Hill}$ during the next 10^4 yr) for 72 effective MCs.

6. The Presence of Mean Motion and Secular Resonances

Among effective MCs we found 45 objects showing MMR and SRs in 20 000 years time-span. Their motion is dominated by close approaches to Mars and various resonances. It is difficult to separate chaotic bahavior caused by resonances and close approaches. However, close to some low-order MMR with Mars we expected to find more chaotic regions of motion, which can be identified by local minima (down-peaks) of LT values. Similar analysis of MMR in the motion of MCs was already presented by Morbidelli and Nesvorny (1999). However, it was based on the sample of 5700 virtual test particles placed on regular grid in semi-major axis, without asteroids close approaching to Mars. We obtained our results using the observational data of known effective MCs population.

The LT values, determined for 45 MCs are presented in Fig. 5. High values of LT (more stable orbits) are in most cases connected with low eccentricity. However, close to various MMR with Mars we found some more chaotic trajectories. Due to the low number of observable effective MCs, it is difficult to identify peaks in LT, connected with particular resonances. We did not find in this sample very chaotic orbits of MCs. Most of them have LT greater than 100 yr.

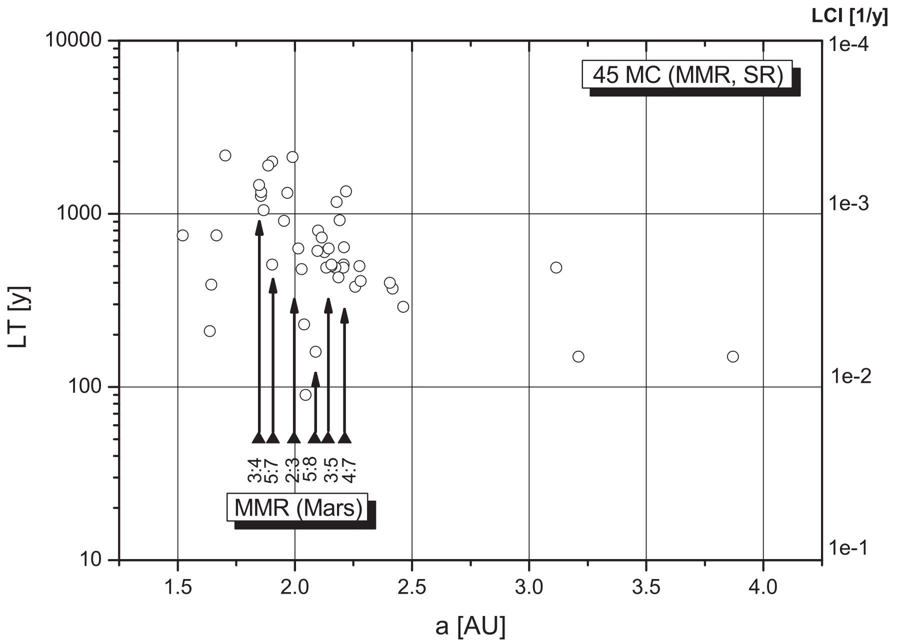

Fig. 5. LT values (*left scale*) and LCIs, (*right*) *vs.* semi-major axis for 45 MCs in MMR and SR. Particular values of a referred to mean motion resonances with Mars are indicated by arrows.

The most important MMR resonances for effective MCs are listed in Table 5. The motion is dominated by various MMR with Mars, but for 7 MCs we found MMR with Jupiter.

As it has been already mentioned, secular resonances with planets were analyzed by observing the nodal and apsidal precession rate of effective MCs and planets. Table 6 shows secular resonances, detected in our effective MCs group.

Most MCs from the selected group exhibit nodal and apsidal resonances with Saturn, Mars and Earth. We also found some apsidal resonances with Mercury and Venus. We did not find any objects in nodal ν_{15} and apsidal ν_5 resonance with Jupiter. The mean nodal $\langle s_5 \rangle$ and apsidal $\langle g_5 \rangle$ precession rate of this planet is much different than typical precession rate of studied asteroids.

For longer timescales, the apsidal resonance with Saturn ν_6 is known as the potential source of chaotic regions in the motion of MCs (Morbidelli and Nesvorny 1999). The region of ν_6 resonance is located close to $a = 2.1$ a.u. (within the inner border of the asteroid belt) and the expected values of LT should be smaller there. For shorter timescales ($\simeq 10^4$ yr) and for a relatively small sample of asteroids this effect is weak or difficult to estimate.

Table 5

The most important MMR resonances (45 selected MCs)

	with Mars
resonance	3:5 MA
	1998 SR96, 2000 KO6, 2000 CB130, 2000 QM130 2002 CT131, 2003 YH115, 2004 EW24
resonance	4:7 MA
	1998 QB1, 1999 NE43, 1999 XT143, 2000 KO6 2001 RQ62, 2002 BJ26, 2003 FM33, 6884 P-L
resonance	2:3 MA
	2000 GO30, 2002 NR2, 2001 VH125, 2002 FH6, 2003 DX18
resonance	3:4 MA
	1994 WB1, 2001 RY17, 2003 JW2, 2003 JP17
resonance	5:7 MA
	1999 VE20, 2002 FV12, 2004 DP5
resonance	5:8 MA
	1999 AA9, 2000 PX5, 2002 EJ77
	Other MMR (Mars):
resonance	1:1 MA
	2003 SC220
resonance	1:2 MA
	1999 KN15, 2000 EL53, 2001 XB124
resonance	1:3 MA
	1999 HX2, 2004 LD5
	Other MMR (Mars)
resonance (asteroid name)	6:7 MA (2001 SX288); 7:8 MA (2002 BK26) 8:9 MA (2002 VQ6), 9:10 MA (2002 UU36)
	MMR with Jupiter
resonance (asteroid name) resonance (asteroid name) resonance (asteroid name)	3:2 (2000 OG44) 4:1 (1998 AK10), (1999 AA9), (1998 YZ21) 7:2 (1995 VM16), (2002 XZ39), (2003 YJ23)

However, MCs with mean apsidal precession closest to $\langle g_6 \rangle$ value locally show a low value of LT (Fig. 6a,b). The mean nodal precession of the object 2001 UW4 is very close to nodal precession of Saturn (Fig. 6b) and the estimated LT is only 270 yr. A similar situation occurs in the case of the nodal precession rate referred to $\langle s_6 \rangle$ value (Fig. 6c,d). The asteroid 1998 YZ21[1] in strong apsidal resonance with Saturn (Fig.6b) has the LT value of 230 yr.

We then conclude that MCs from our sample, located close to the region of strong secular resonances with Saturn, have more chaotic trajectories.

There are also two objects with small LT values in this sample. 2002 EJ77 (LT=160 yr) is in weak nodal resonance with Mars and MMR 5:8 with Mars. 1998

[1]The orbital data of this asteroid were significantly revised due to new observational data. The alternate designation is 2002 UE41.

Table 6

Apsidal and nodal resonances in the motion of 45 selected MCs

Nodal resonance		Apsidal resonance	
1994 WB1	ν_{13},ν_{16}	1998 SR96	ν_6
1999 LX3	ν_{13}	1995 VM16	ν_6
2000 GO30	ν_{13}	1998 QB1	ν_6
1999 XO141	ν_{16}	1998 AK10	ν_6
2001 FO86	ν_{13},ν_{14}	1999 NE43	ν_6
2001 RY17	ν_{13},ν_{14}	1998 YZ21	ν_6
2001 UT17	ν_{16}	1999 XT143	ν_6
2001 UW4	ν_{16}	2000 KO6	ν_6
2001 SX288	ν_{13}	2000 GO30	ν_6
2002 NR2	ν_{16}	1999 X0141	ν_3,ν_4
2002 BK26	ν_{13},ν_{14}	2000 CB130	ν_6
2002 EJ77	ν_{13}	2000 PX5	ν_6
2002 FH6	ν_{16}	2000 QM130	ν_3,ν_4
2002 AZ2	ν_{16}	2001 UT17	ν_2
2002 FV12	ν_{16}	2001 RQ62	ν_6
2003 EF16	ν_{16}	2001 VH125	ν_6
2003 JP17	ν_{13}	2002 BK26	ν_2
2003 JW2	ν_{13},ν_{14}	2002 FH6	ν_1
2002 VQ6	ν_{13},ν_{14}	2002 AZ2	ν_3,ν_4
2003 CP	ν_{13},ν_{14}	2003 DX18	ν_6
2003 JP17	ν_{13},ν_{14}	2003 EF16	ν_3,ν_4
2004 DP5	ν_{16}	2003 MA7	ν_6
		2002 VQ6	ν_3,ν_4
		2003 SC220	ν_2
		2004 EW24	ν_6
		2004 DP5	ν_3,ν_4

Remarks:

ν_{13} – nodal resonance with Mars	ν_1 – apsidal resonance with Mercury
ν_{14} – nodal resonance with Earth	ν_2 – apsidal resonance with Venus
ν_{16} – nodal resonance with Saturn	ν_3 – apsidal resonance with Earth
	ν_4 – apsidal resonance with Mars
	ν_6 – apsidal resonance with Saturn

AK10 (LT=90 yr) indicates weak apsidal resonance with Saturn and MMR 4:1 with Jupiter. Their chaotic behavior is caused by the combination of various mean motion and secular resonances.

7. Summary and Conclusions

The motion of effective MCs is dominated by close approaches and weak resonances with Saturn, inner planets and Jupiter. The number of asteroids potentially hazardous to Mars in the close future is low (72 objects, *i.e.*, about 2%). The mean

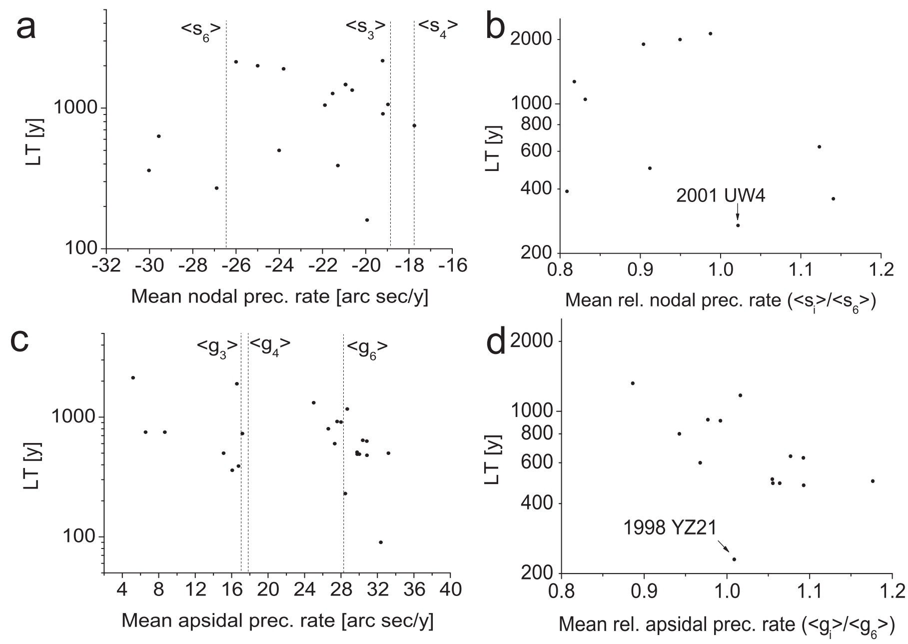

Fig. 6. The LT values *vs.* mean precession rate of MCs in nodal (*a,b*) and apsidal (*c,d*) resonance. *a* – LT and mean nodal rate of asteroids, compared to mean nodal precession rate of three planets ($\langle s_3 \rangle$ – Earth nodal rate, $\langle s_4 \rangle$ – Mars nodal rate, $\langle s_6 \rangle$ – Saturn nodal rate), *b* – LT and mean relative nodal rate of asteroids (divided by Saturn $\langle s_6 \rangle$ rate), *c* – LT and mean apsidal rate of asteroids compared to mean apsidal precession rate of planets ($\langle g_3 \rangle$ – Earth apsidal rate, $\langle g_4 \rangle$ – Mars apsidal rate, $\langle g_6 \rangle$ – Saturn apsidal rate), *d* – LT and mean relative apsidal rate of asteroids divided by Saturn $\langle g_6 \rangle$ rate. Two asteroids in strong nodal/apsidal SR with Saturn are also shown (*b,d*).

approach frequency to Mars within the $3R_{\rm Hill}$ distance is equal to only a few times per 1 000 yr. Most of the determined LT values are of the order of 100 yr. There is a kind of qualitative relationship between the mean approach frequency and LT values: for higher approach frequencies we obtain smaller LT. This relationship is not a quantitative, universal law because LT values are not always connected with macroscopic dynamical lifetimes of minor bodies (Morbidelli and Froeschle 1996). We found a group of more chaotic MCs (LT $<$ 100 yr), existing probably in a different, more unstable dynamical regime.

Another mechanism, having a possible influence on MCs chaoticity are MMR with Mars (Morbidelli and Nesvorny 1999) and Jupiter.

The third possible cause of low LT values are the secular resonances with Saturn. Other secular resonances with Mars and Earth are also present, but they do not play any important role.

All results were obtained by numerical simulation of a limited sub-population of MCs. Similar conclusions concerning the effect of inner planets and Saturn resonances were presented by (Morbidelli and Nesvorny 1999), but a direct comparison is not possible due to the different timescale and class (inner belt) of studied objects.

As far as the motion of asteroids potentially hazardous to Mars is concerned, we confirmed the presence of three main culprits that cause chaoticity: regular close approaches, mean motion resonances with Mars and secular resonances with Saturn. For some of the studied object, these effects occur simultaneously and are difficult to separate.

Due to the chaotic behavior of effective MCs, it is difficult to obtain reliable predictions of their future orbital evolution and the risk of potential collision. Most of the studied MCs have been discovered over the last few years, hence their orbital elements have only been determined with limited precision. In the nearby future we expect, though, a systematic growth of the known population of effective MCs and the application of new observational results.

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